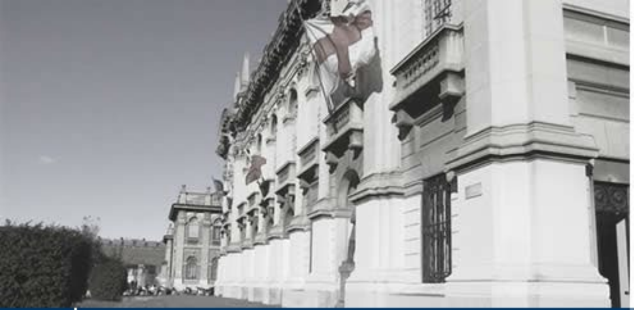


Advanced course on

HIGH RESOLUTION ELECTRONIC MEASUREMENTS IN NANO-BIO SCIENCE

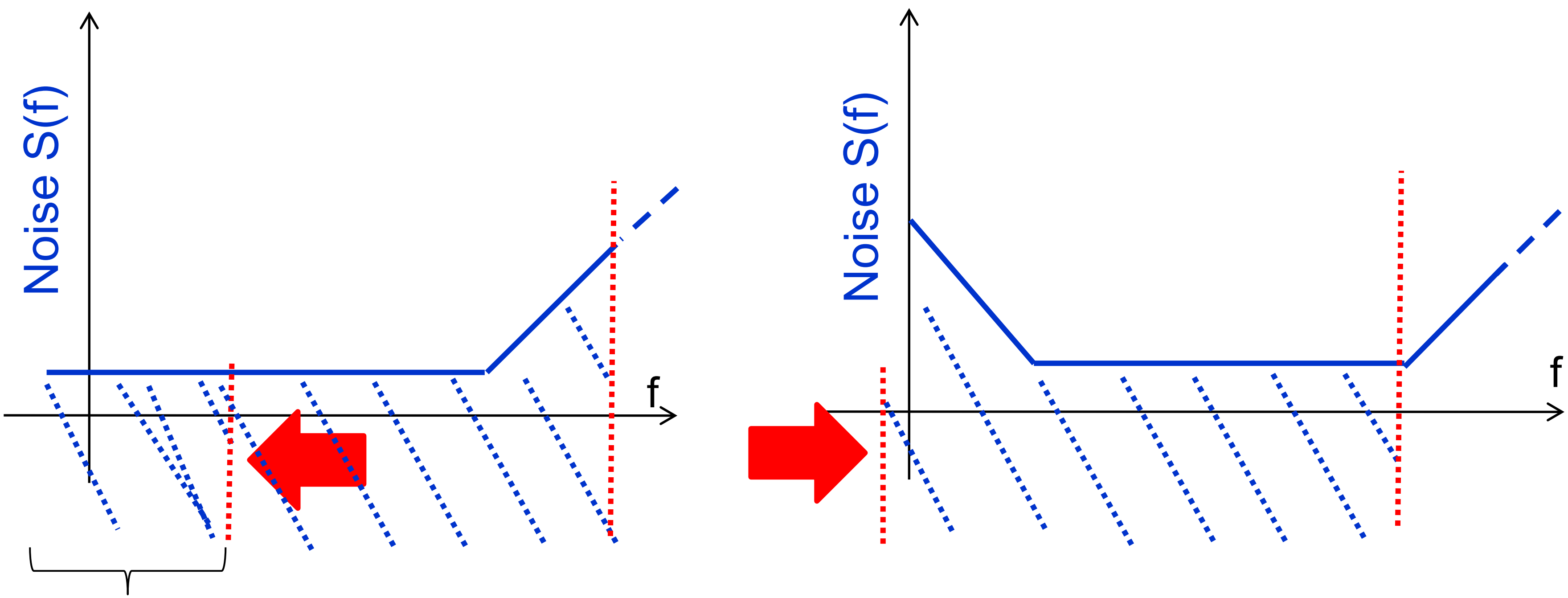
POLITECNICO DI MILANO

Measurements at a given frequency

The Lock-in concept

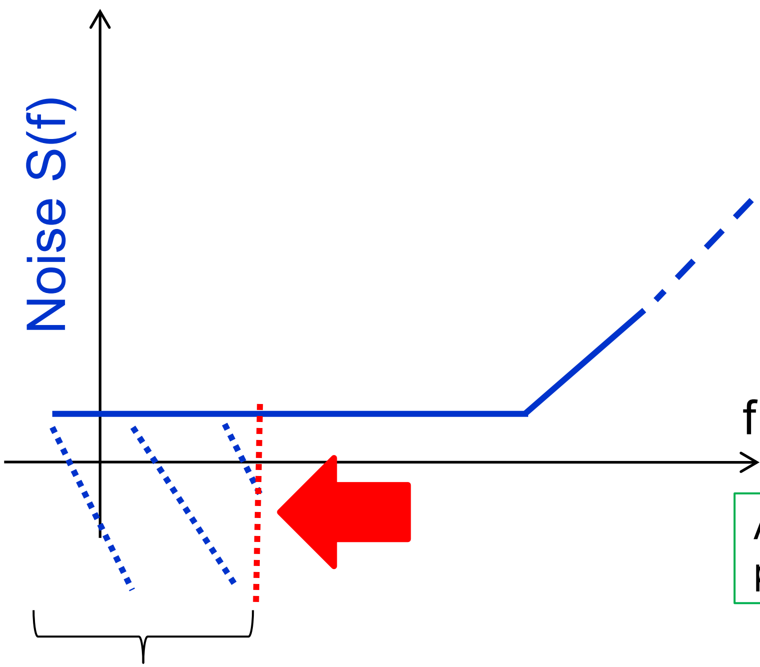
Marco Sampietro

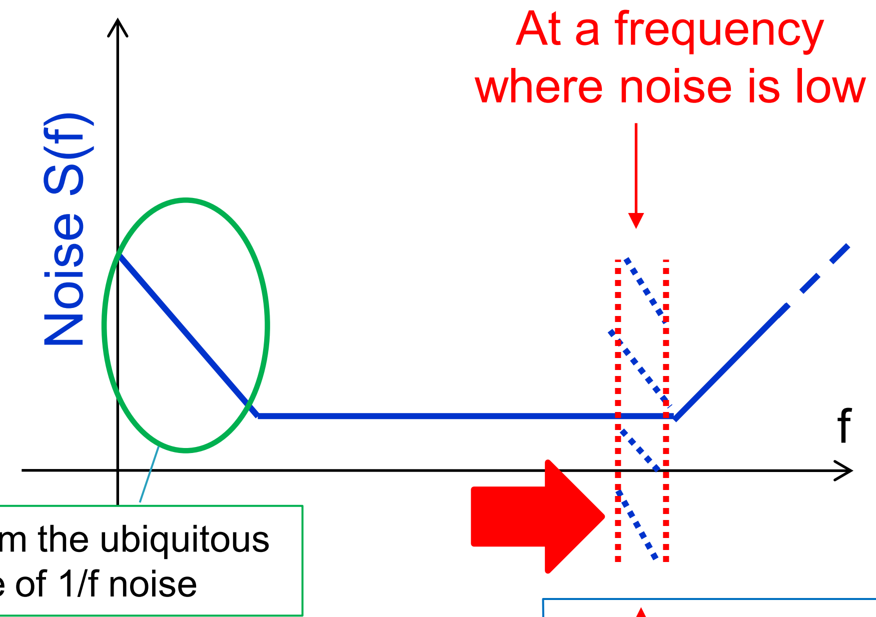
Small bandwidth ⇒ Low noise



Small BW ➡ small noise

… small bandwidth at high frequency





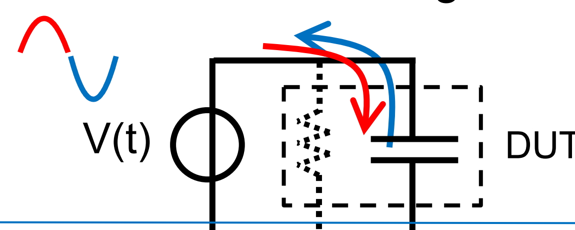
Small BW → small noise

Small BW → small noise

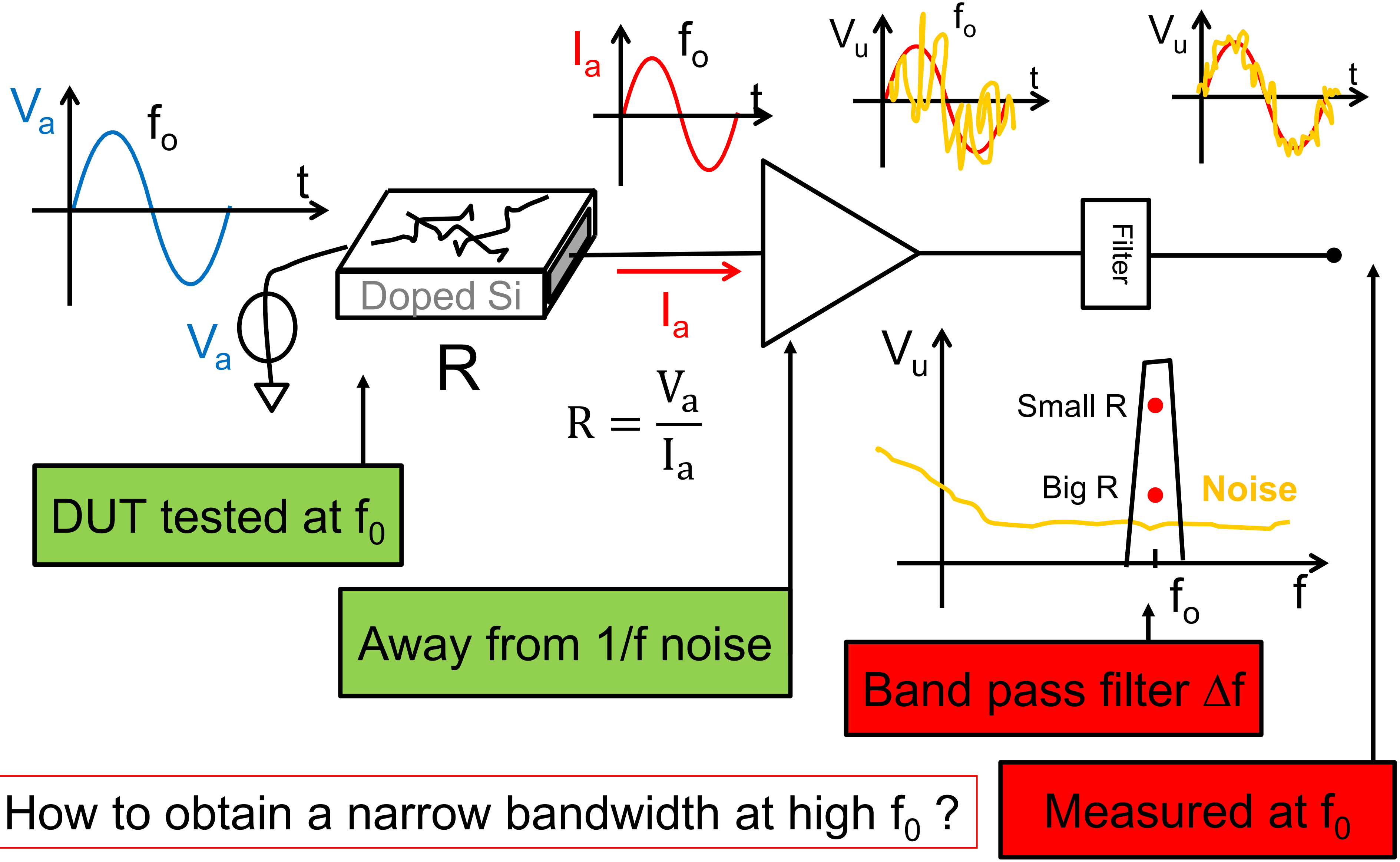
Signal is here

A repeated measurement at high frequency needs “**RECHARGEABLE**” events (AC electronic devices, reversible redox, pump&probe, …)

Ex. : Induced charge on metal plates



Example : measurement of R

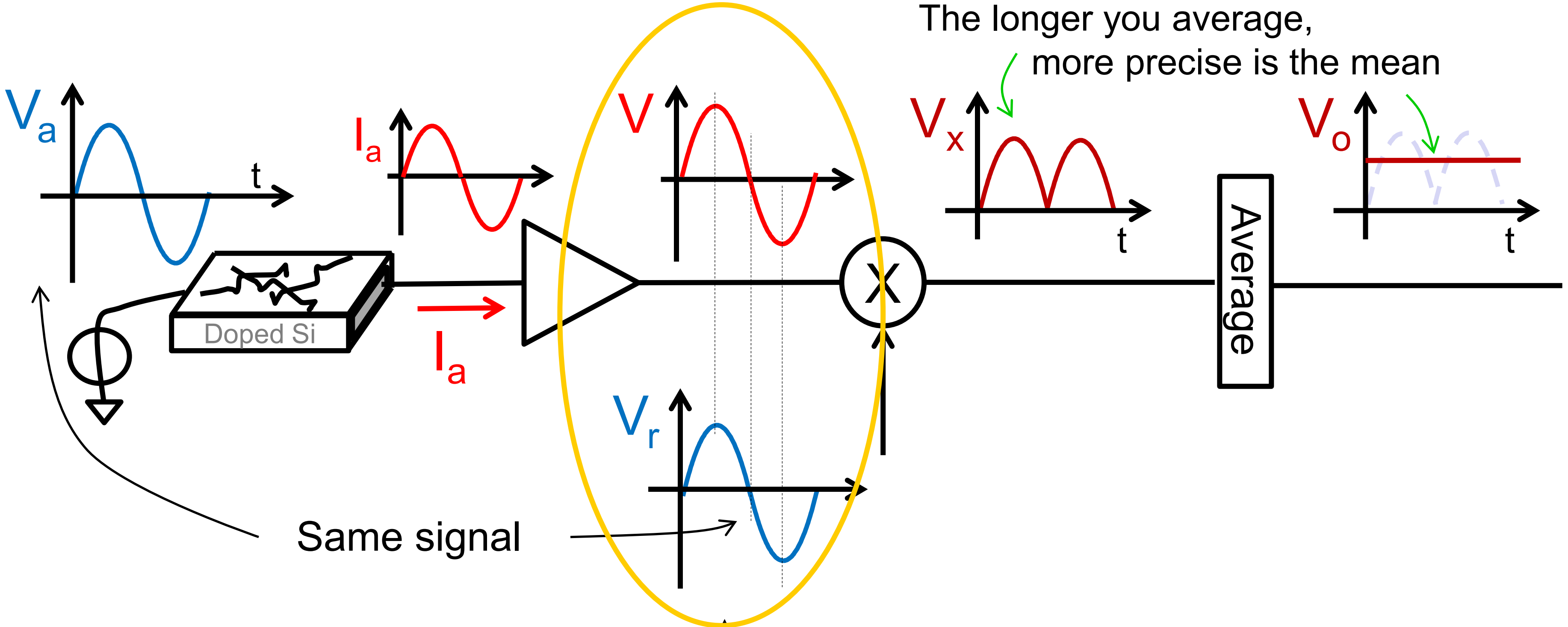


How to obtain a narrow bandwidth at high f_0 ?

Outline of the lesson

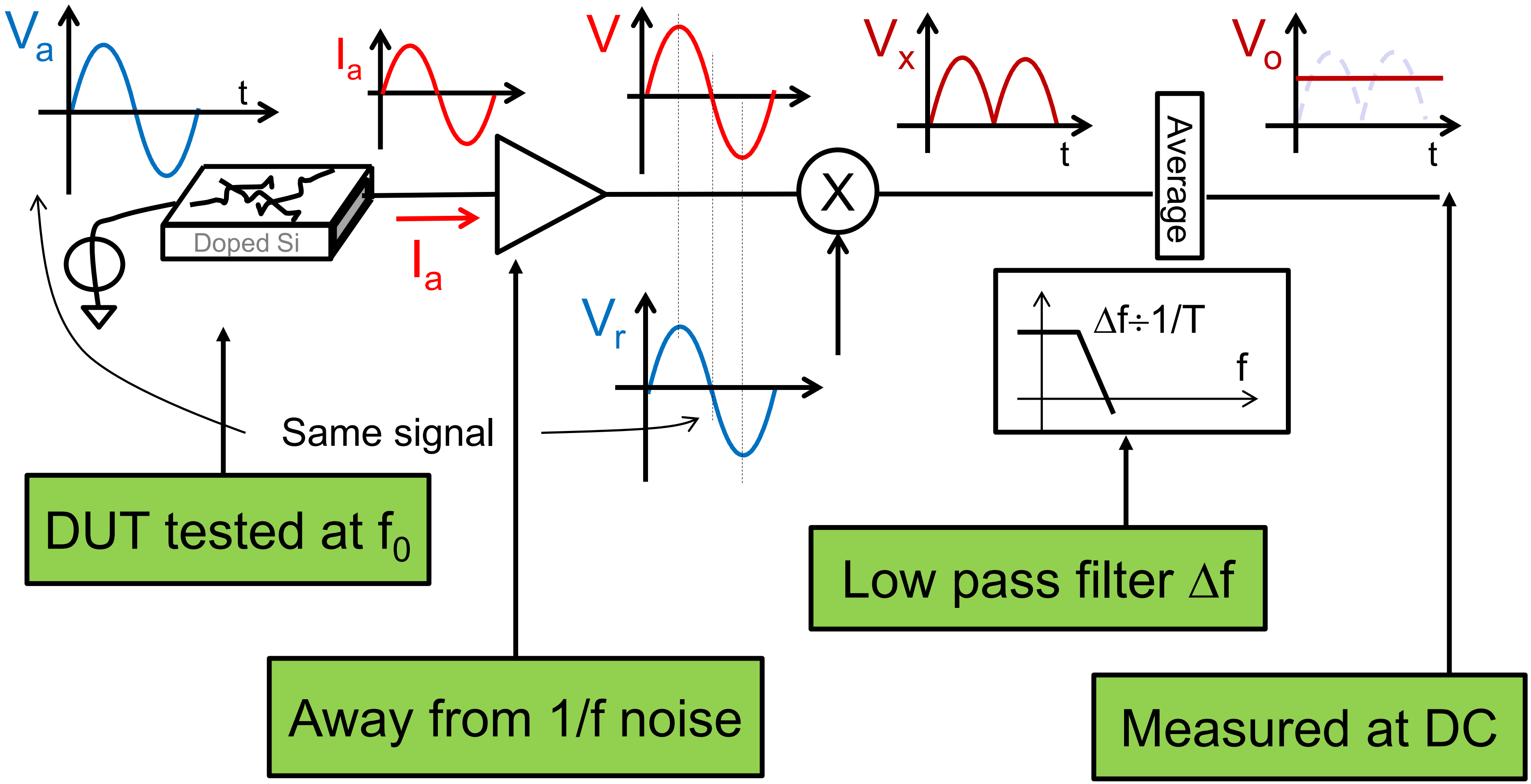
- The Lock-in concept 20 min
- Performance in sensitivity 20 min

The LOCK-IN idea



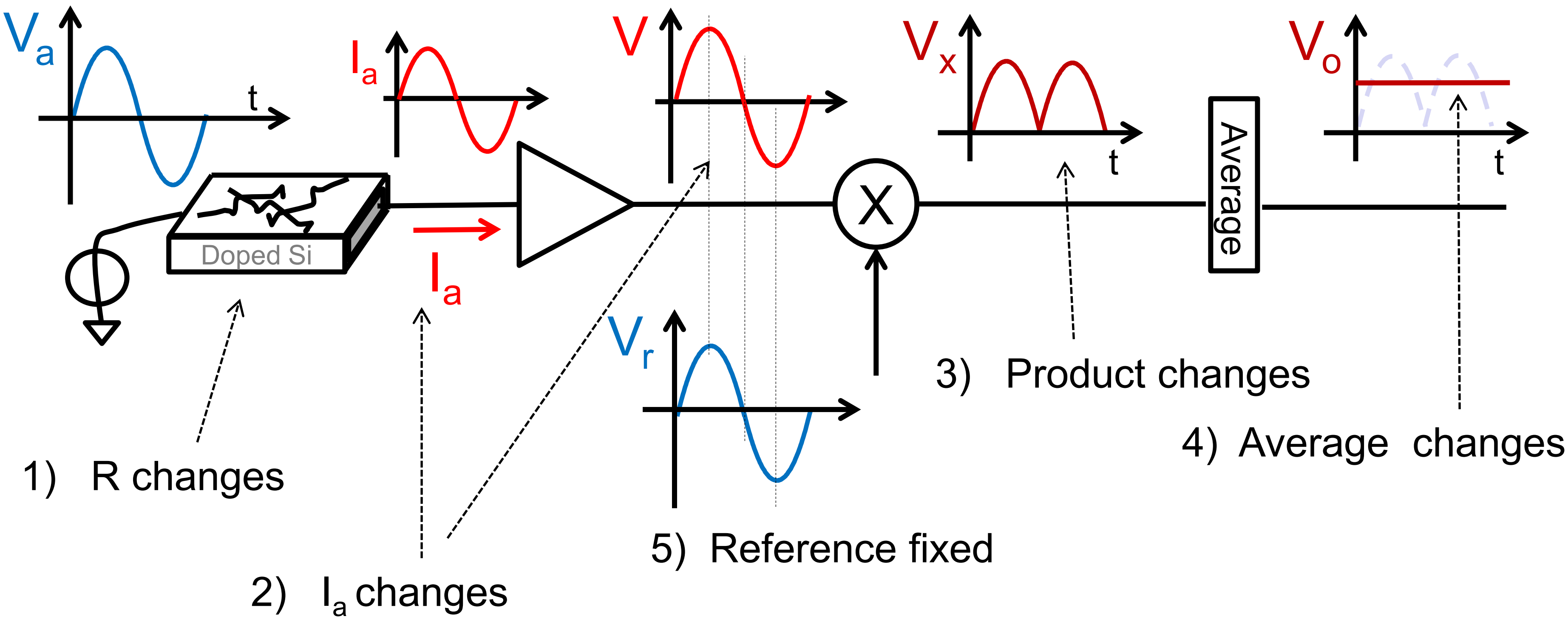
Same frequency and phase
(*coherent detection*)

The LOCK-IN idea



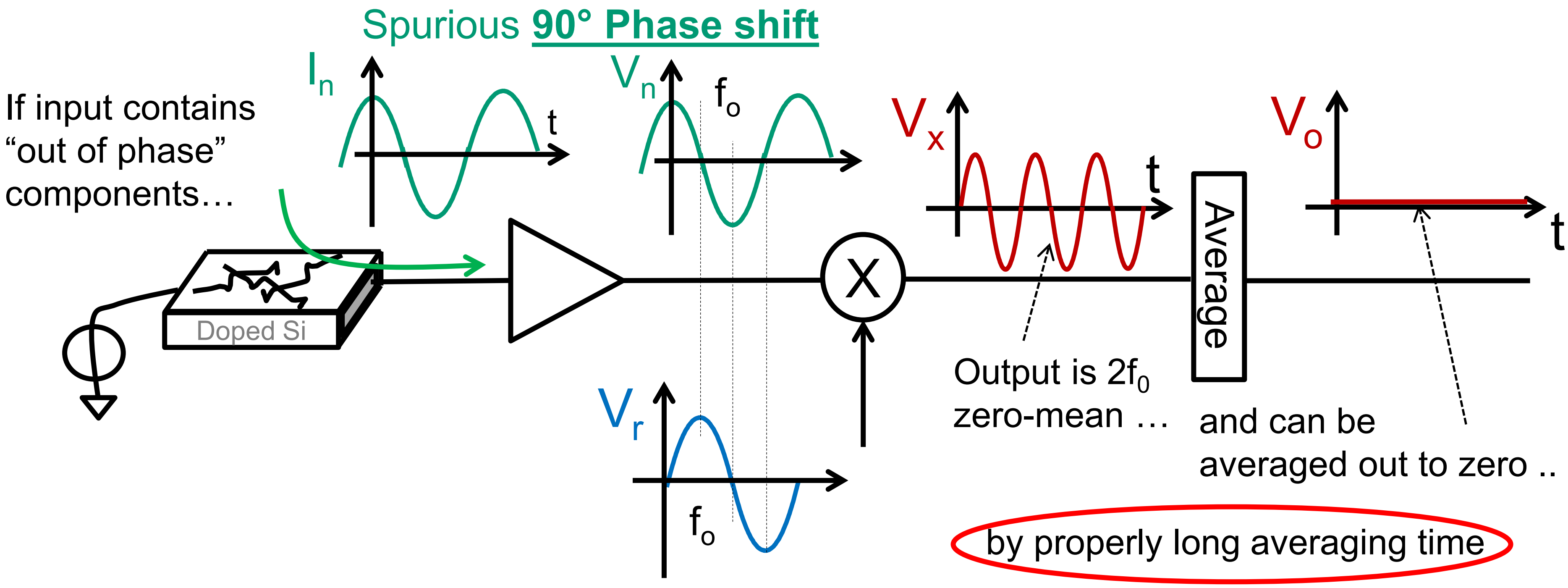
Credited to Robert Dicke, founder of Princeton Applied Research (PAR) in the 1960's.

Tracking sensor with the LOCK-IN



DUT variations can be tracked with time by simply monitoring the level of the output

Phase selectivity of the Lock-in



Small bandwidth → Long averaging time → High rejection of «spurious»

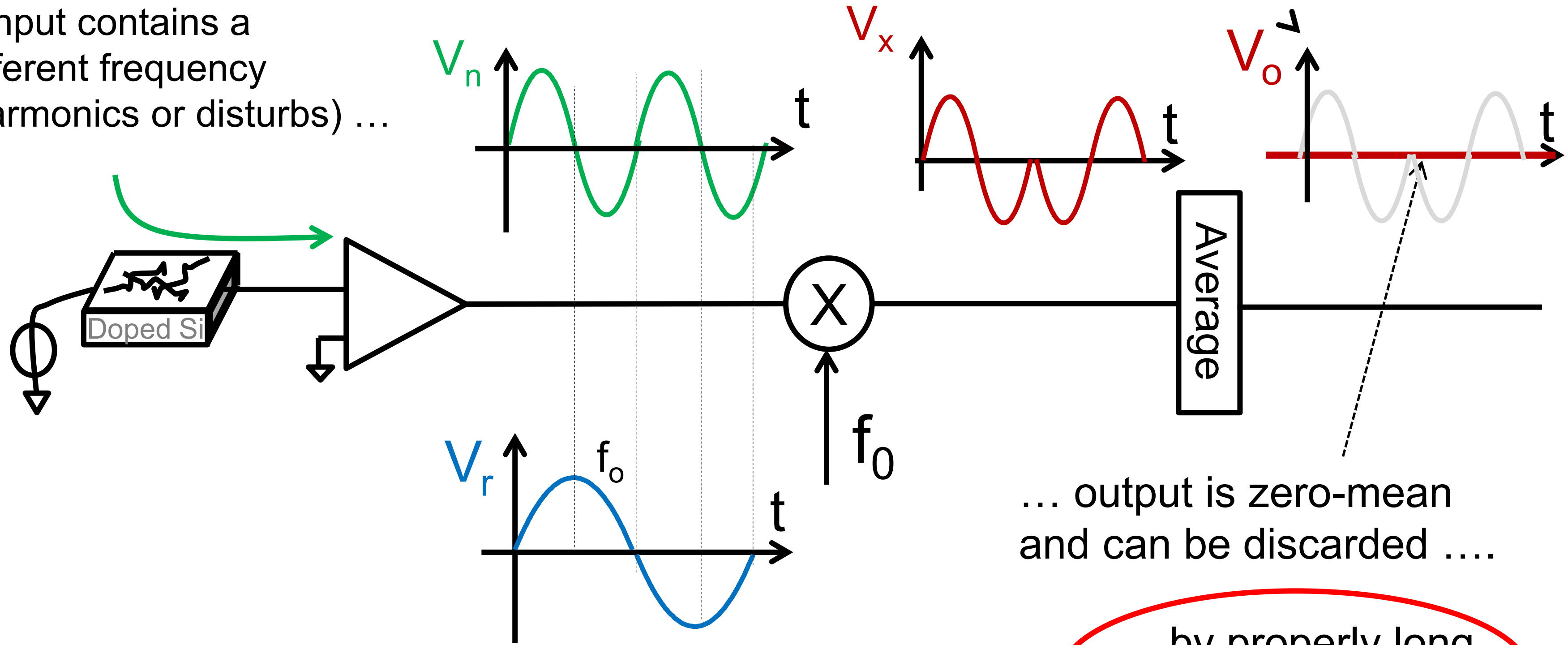
Long measurement time

Low noise

Signals with 90° ***phase-shift*** to reference are rejected

Frequency selectivity of the Lock-in

If input contains a different frequency (harmonics or disturbs) …

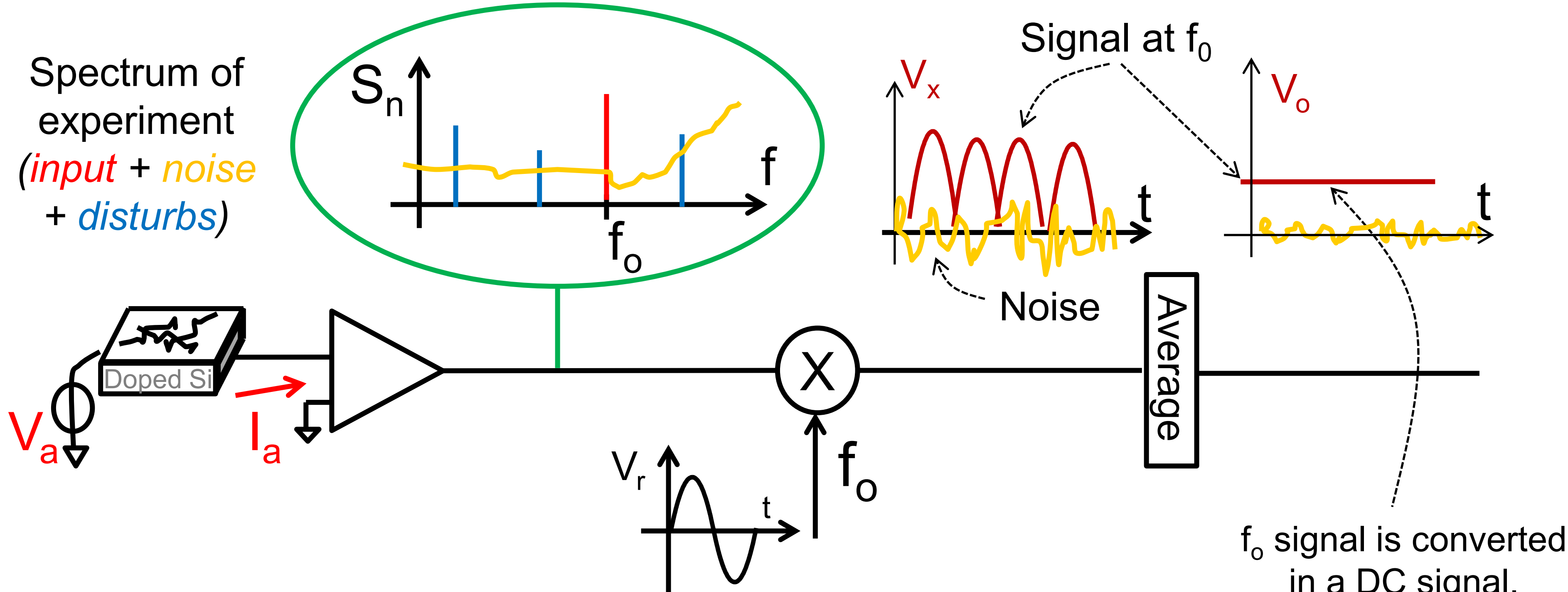


… output is zero-mean and can be discarded ….

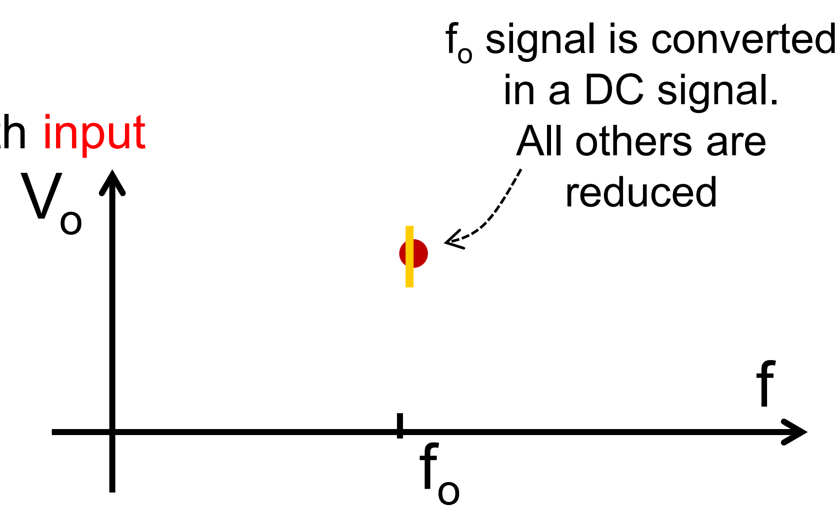
… by properly long averaging

Signals with different frequency to reference are rejected

Noise suppression



Reference in phase with input

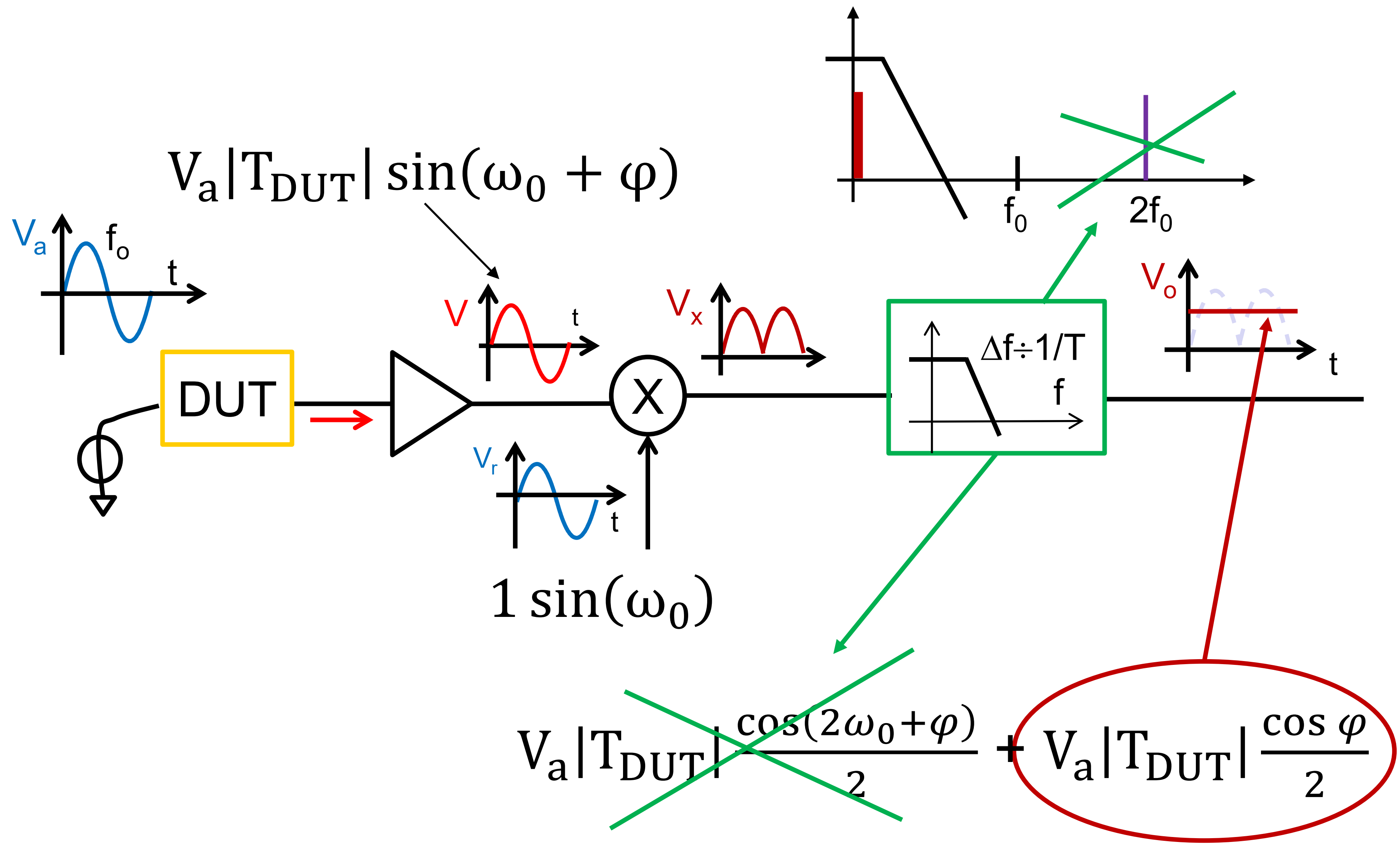


NOISE is REDUCED

provided a long time for averaging

SIGNAL is EXTRACTED

Analytical view - Signal

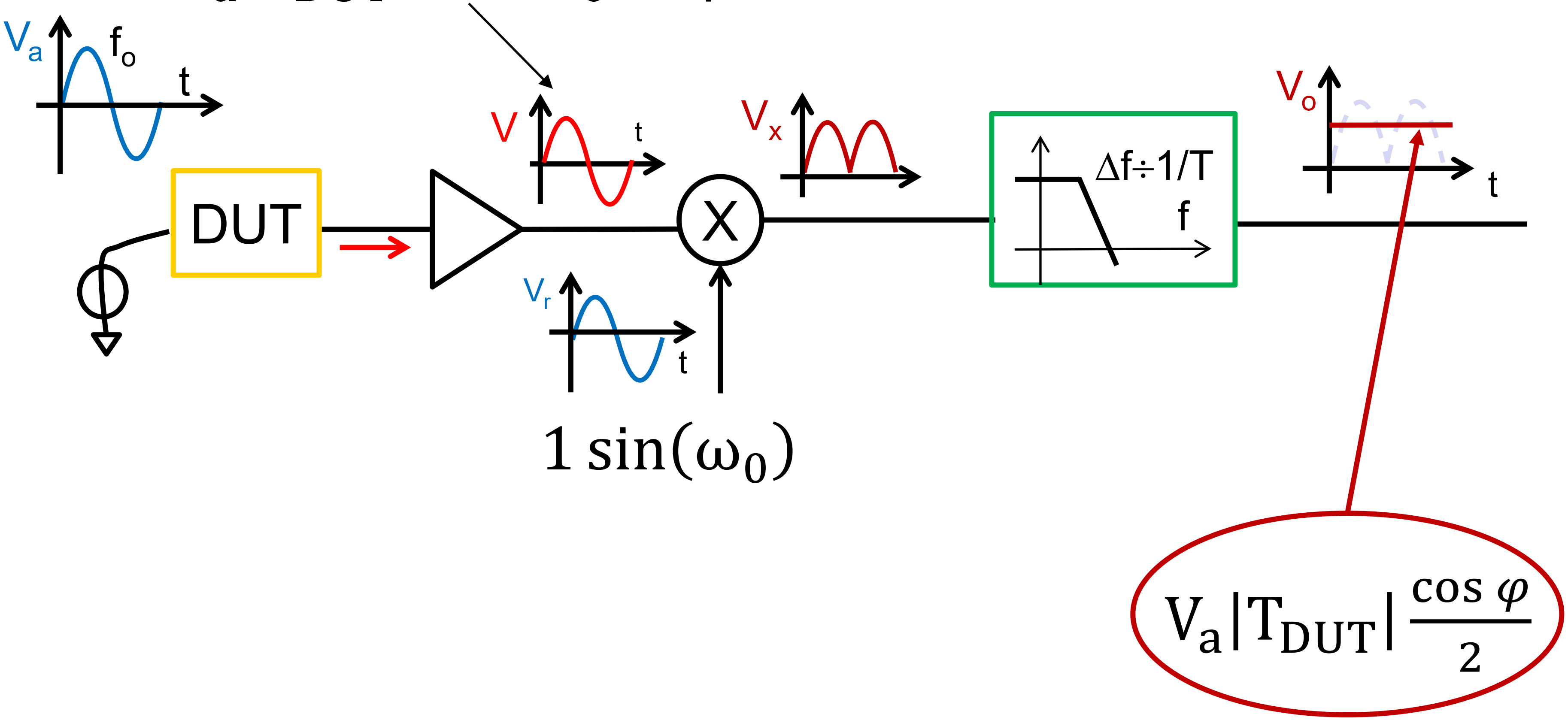


Analytical view - Power

$$P_{in} = \frac{(V_a|T_{DUT}|)^2}{2}$$

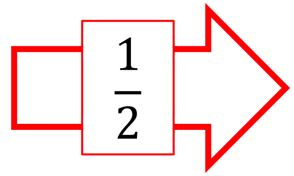
$$P_{out} = \left(V_a|T_{DUT}|\frac{\cos\varphi}{2}\right)^2$$

$$V_a|T_{DUT}|\sin(\omega_0 + \varphi)$$

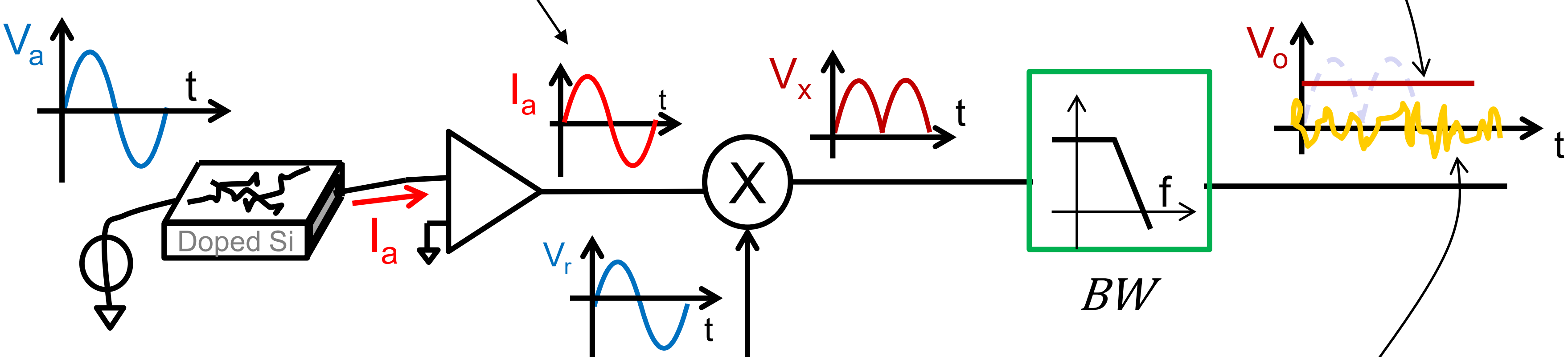


Signal to Noise RATIO

$$P_{in|signal} = \frac{(V_a|T_{DUT}|)^2}{2}$$



$$P_{out|signal} = \left(V_a|T_{DUT}|\frac{\cos\varphi}{2}\right)^2$$



$$1\,\sin(\omega_0)$$

$$P_{out|noise} = S_{in|noise}(f_0)\cdot\frac{1}{2}\cdot BW$$

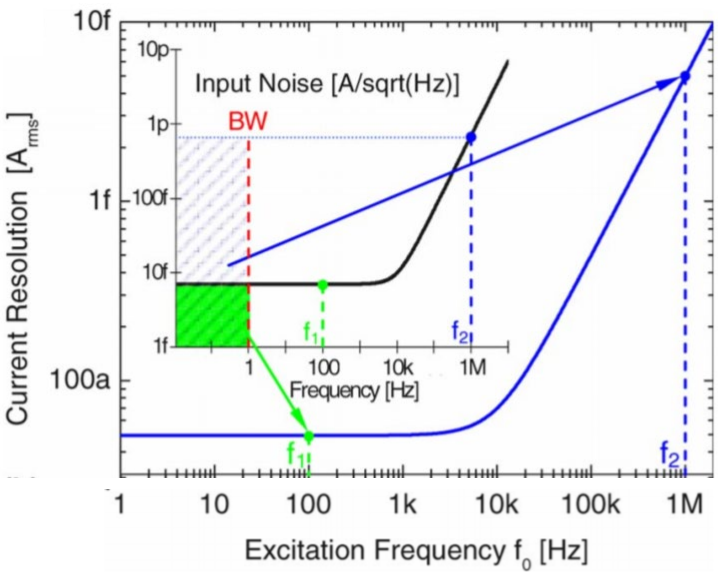
$$\frac{P_{in|signal}}{P_{in|noise}} = \frac{P_{in|signal}\cdot\cancel{\frac{1}{2}}}{S_{in|noise}(f_0)\cdot\cancel{\frac{1}{2}}\cdot BW} = \frac{P_{out|signal}}{P_{out|noise}}$$

same BW of the lock-in around f_0

The Lock-in can be considered an optimal detector of AC signals

Lock-In Noise Filtering

$$\frac{P_{in|signal}}{P_{in|noise}} = \frac{P_{in|signal}}{S_{in|noise}(f_0) \cdot BW}$$



- The modulation *whiten* the noise spectrum in base band at the value sampled at f_0
- The order of the filter is not critical for the noise (a first order is ok even with non-white noise)
- The filter has to properly cut the $2f_0$ component

M. Carminati et al. “Attofarad resolution potentiostat for electrochemical measurements on nanoscale biomolecular interfacial systems.,” Rev. Sci. Instrum., vol. 80, no. 12, p. 124701 (2009), doi: 10.1063/1.3245343.

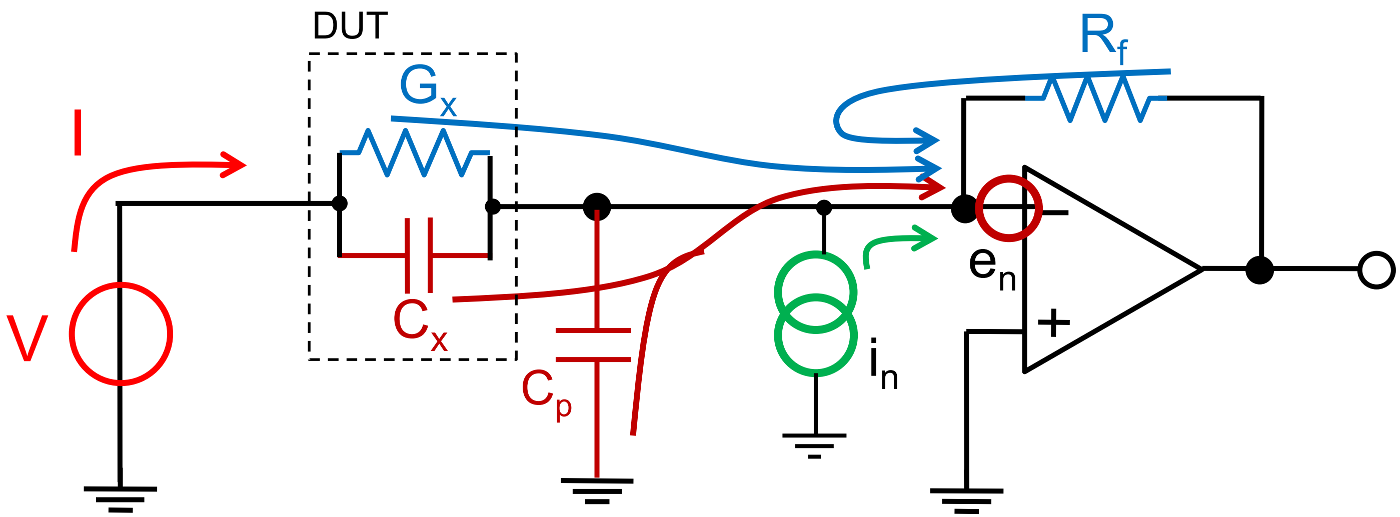
Outline of the lesson

- The Lock-in concept 20 min

- Performance in sensitivity 20 min

Limit of sensitivity : Noise Analysis

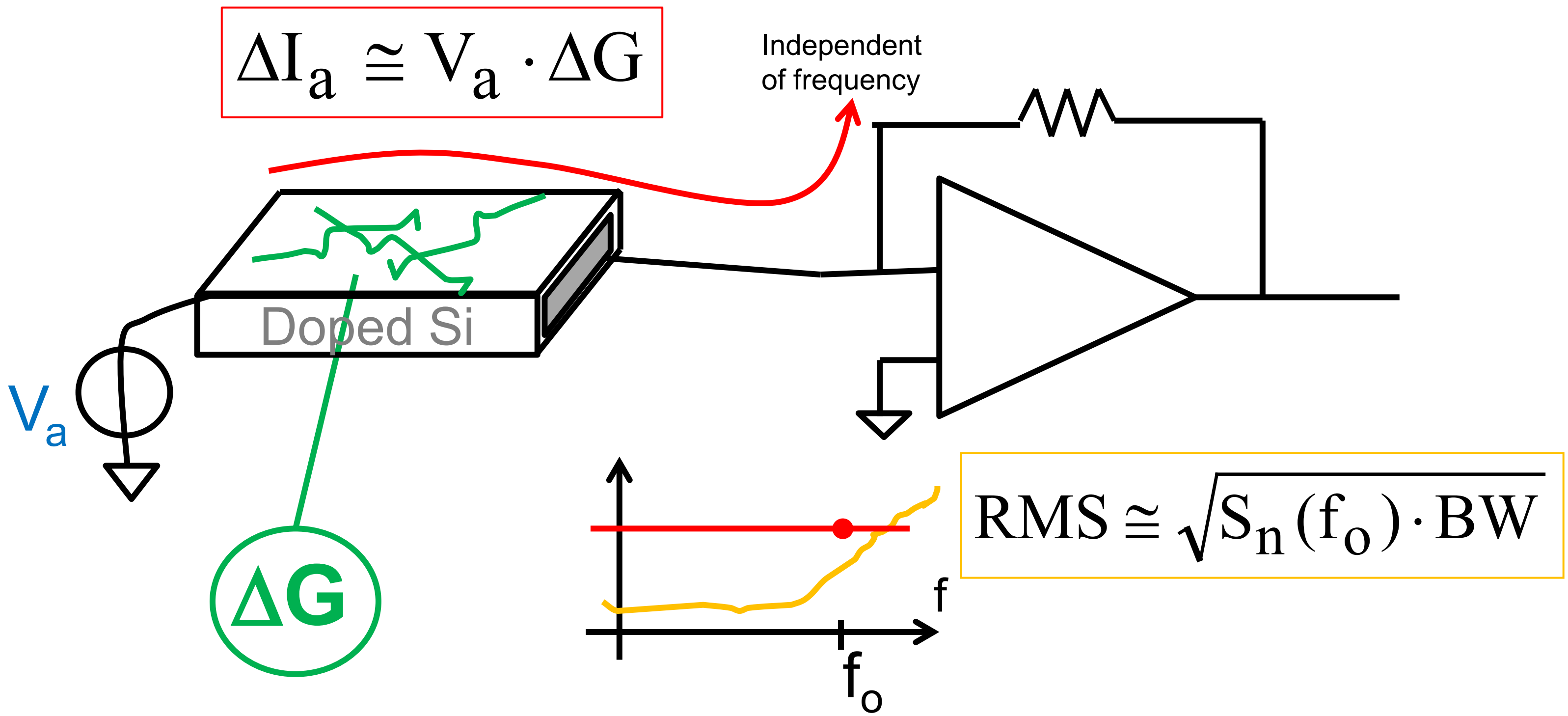
A recap of the input noise of the TIA :



$$S_i = \overline{i_n^2} + 4kT(G_x + G_f) + \overline{e_n^2}\omega^2\left(C_x + C_p\right)^2 + \overline{e_n^2}(G_x + G_f)^2$$

To be compared with the Signal (V, I)

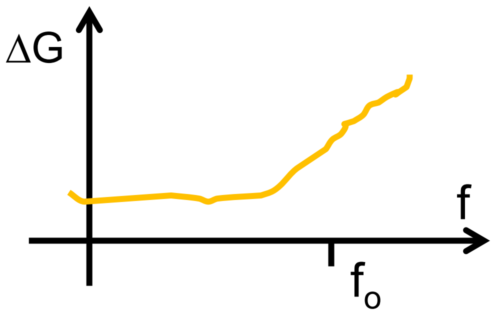
Limit of sensitivity - for R



The limit of detection is reached when

$$\Delta I_a \cong RMS$$

$$\Delta G \cong \frac{\sqrt{S_n(f) \cdot BW}}{V_a}$$

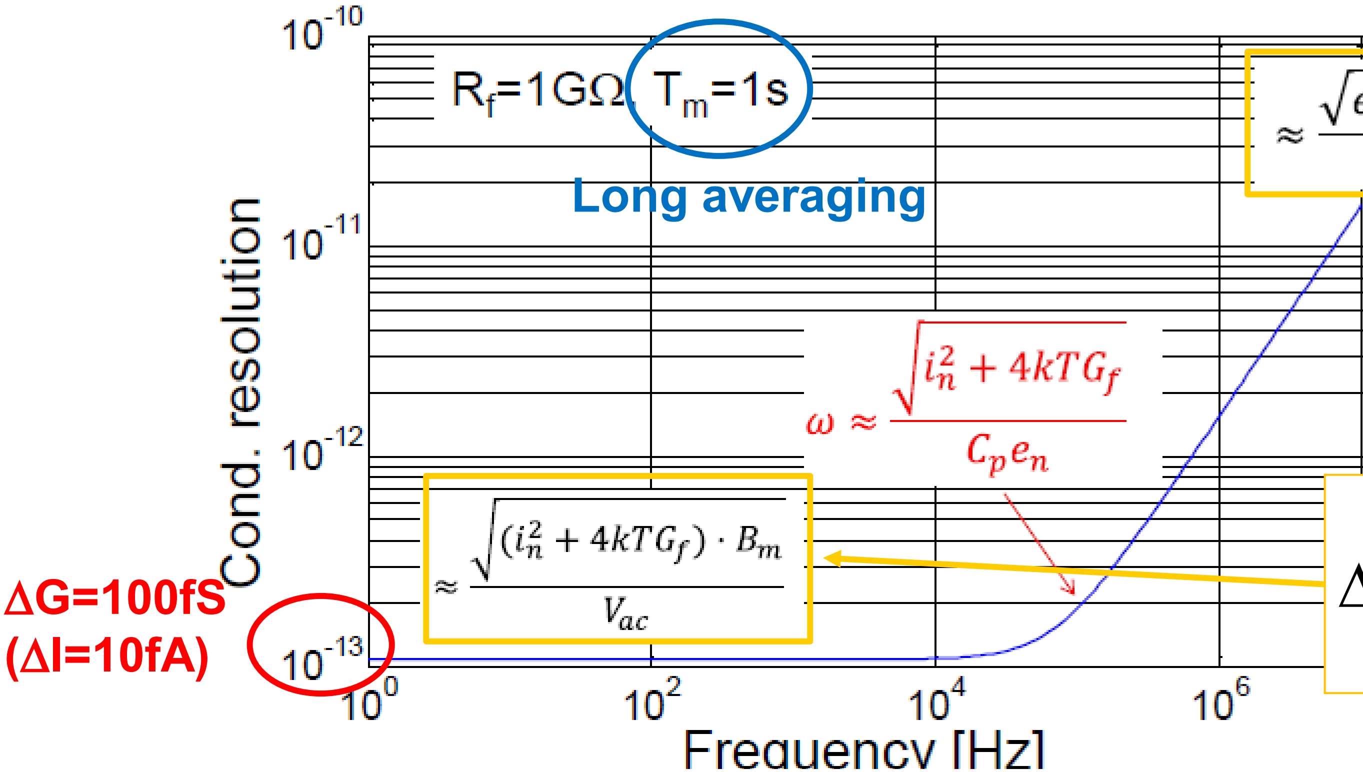


Limit of sensitivity - for R

Still margin of improvement !

$$e_n = \frac{5nV}{\sqrt{Hz}}, i_n = \frac{10fA}{\sqrt{Hz}}, V_{ac} = 100mV, C_p = 5pF, C_x \ll C_p$$

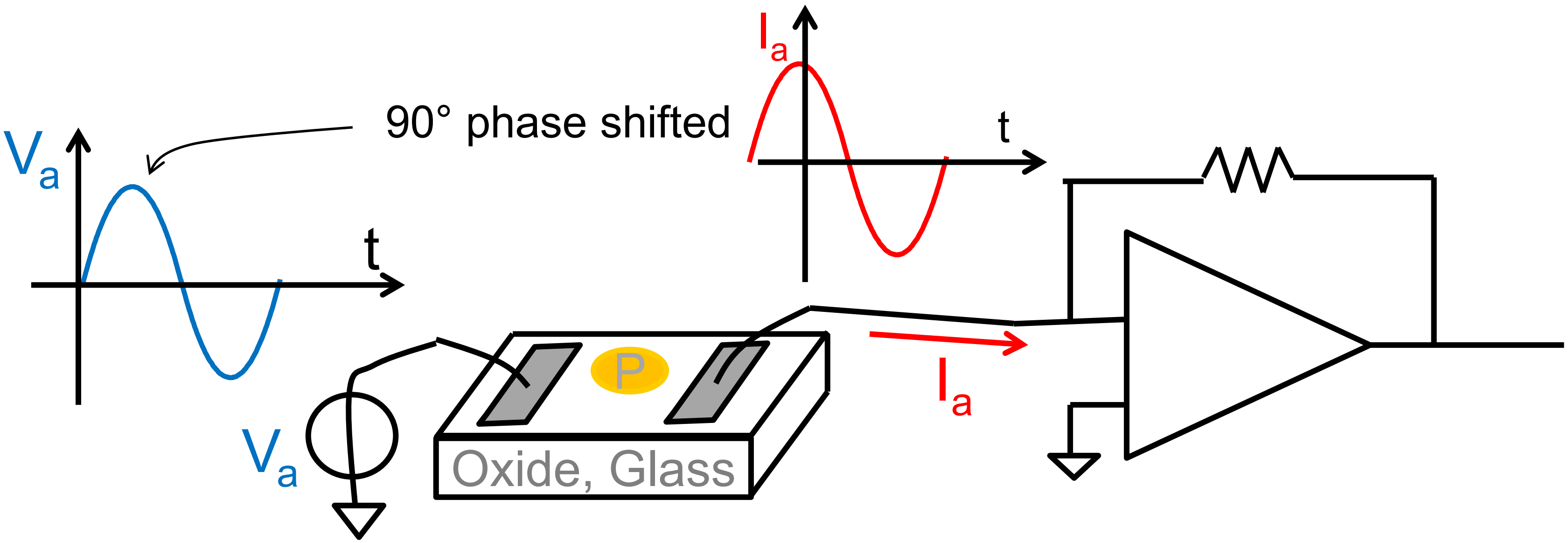
Good electronics **Good mounting**



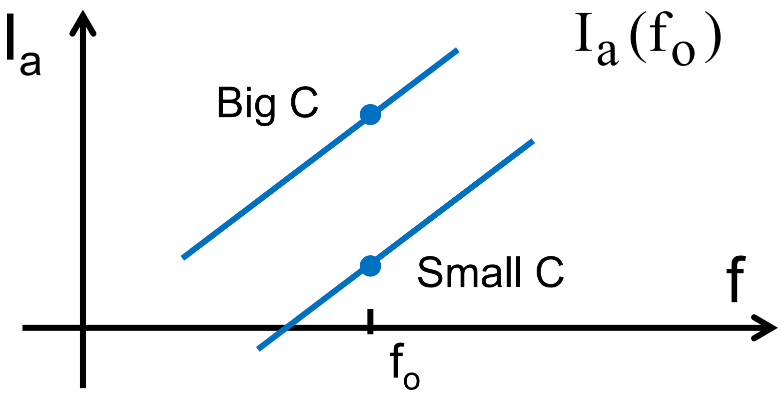
$$\Delta G \cong \frac{\sqrt{S_n(f) \cdot BW}}{V_a}$$

$$S_i = \overline{i_n^2} + 4kT(G_x + G_f) + \overline{e_n^2}\omega^2 (C_x + C_p)^2 + \overline{e_n^2}(G_x + G_f)^2$$

Sinusoidal measurement of C

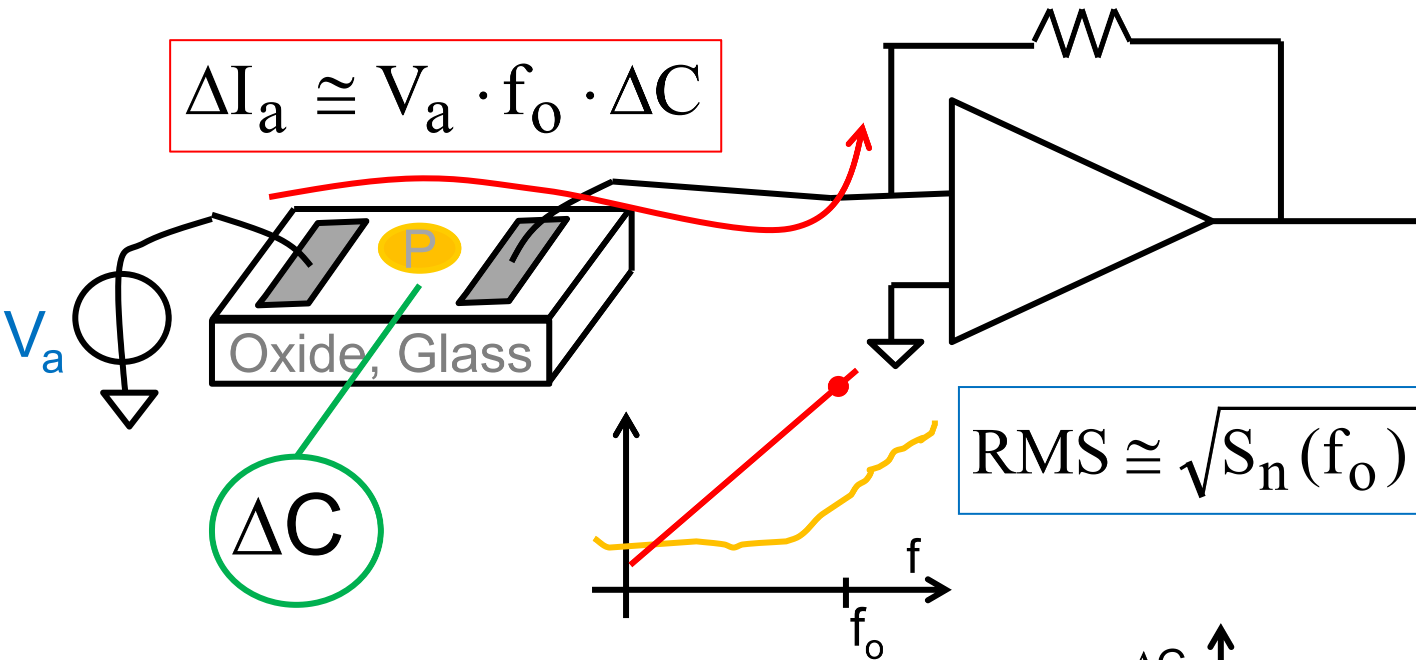


$$\frac{V_a(f_o)}{I_a(f_o)} = \frac{1}{j2\pi f_o \cdot C}$$



Limit of sensitivity - for C

$$\Delta I_a \cong V_a \cdot f_o \cdot \Delta C$$

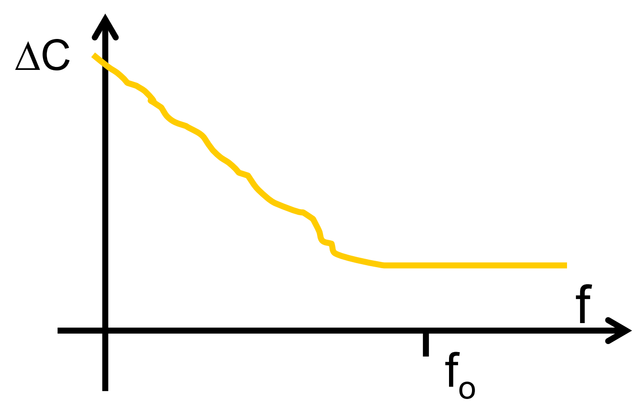


$$RMS \cong \sqrt{S_n(f_o) \cdot BW}$$

Limit when

$$\Delta I_a \cong RMS$$

$$\Delta C \cong \frac{\sqrt{S_n(f) \cdot BW}}{V_a \cdot f}$$



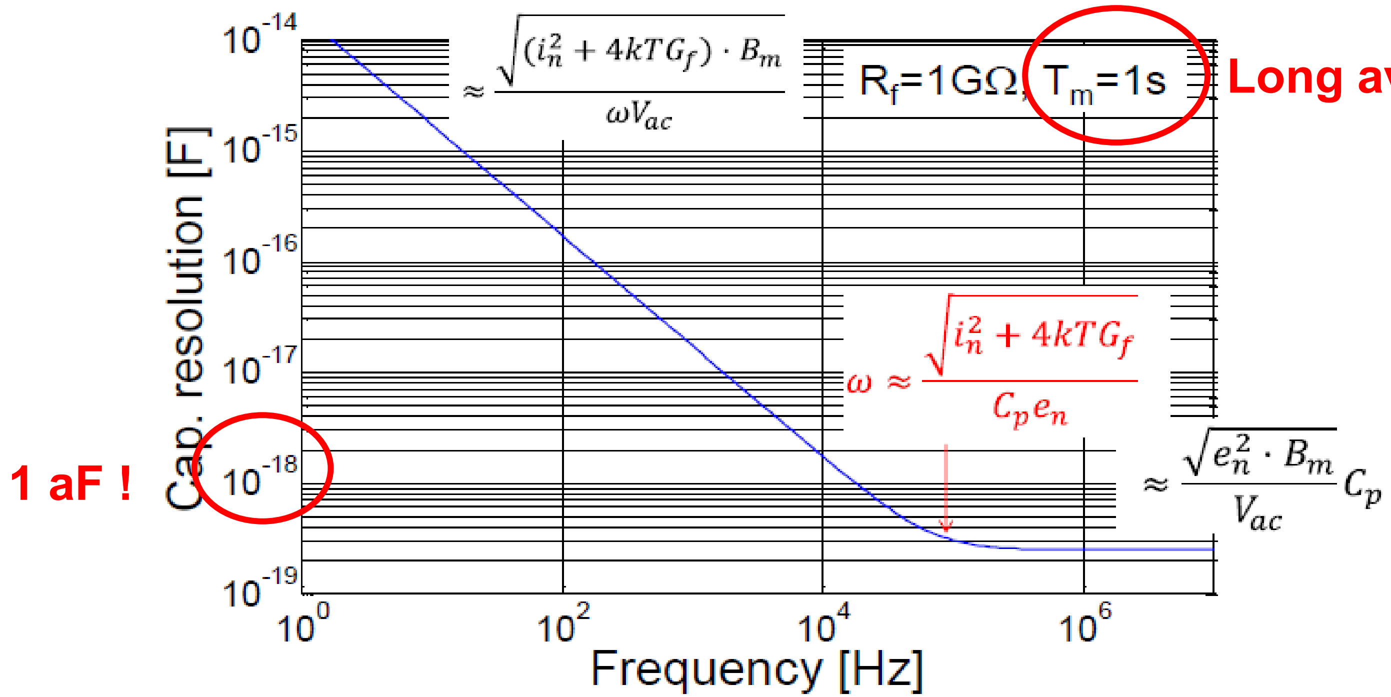
Limit of sensitivity - for C

Still margin of improvement (see next slide) !

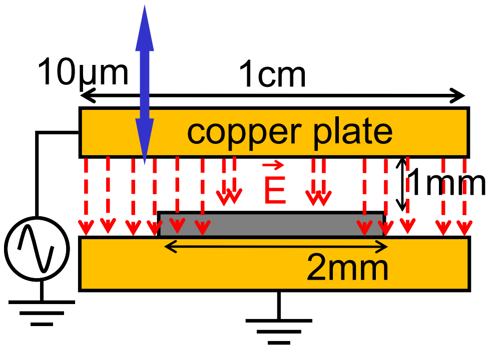
$$e_n = \frac{5nV}{\sqrt{Hz}}, i_n = \frac{10fA}{\sqrt{Hz}}, V_{ac} = 100mV, C_p = 5pF, C_x \ll C_p$$

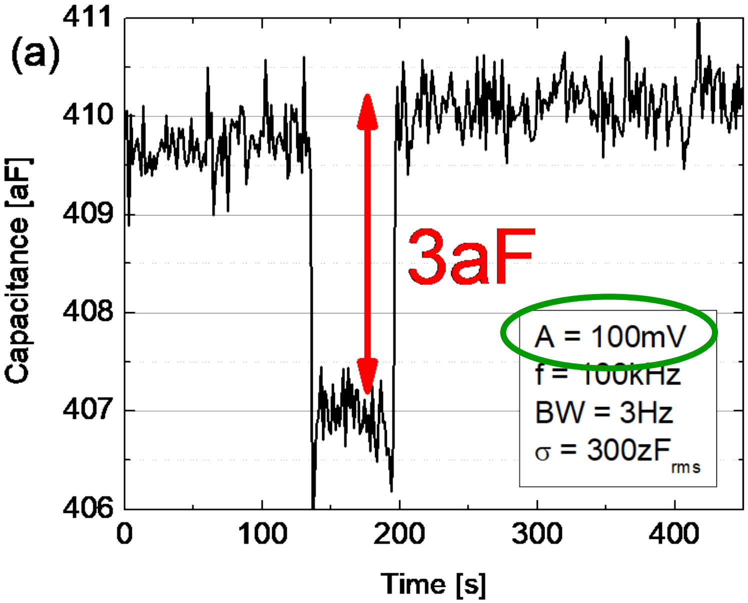
Good electronics

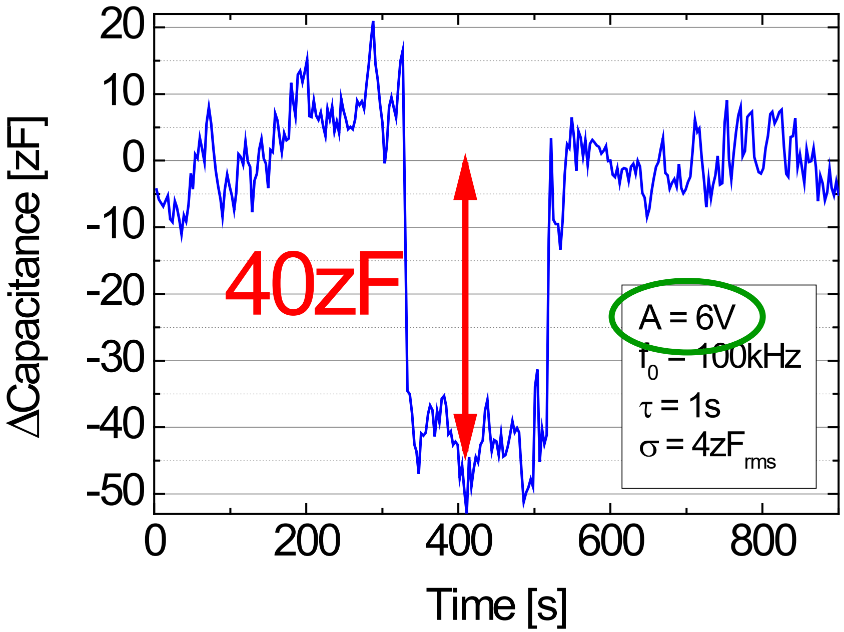
Good mounting



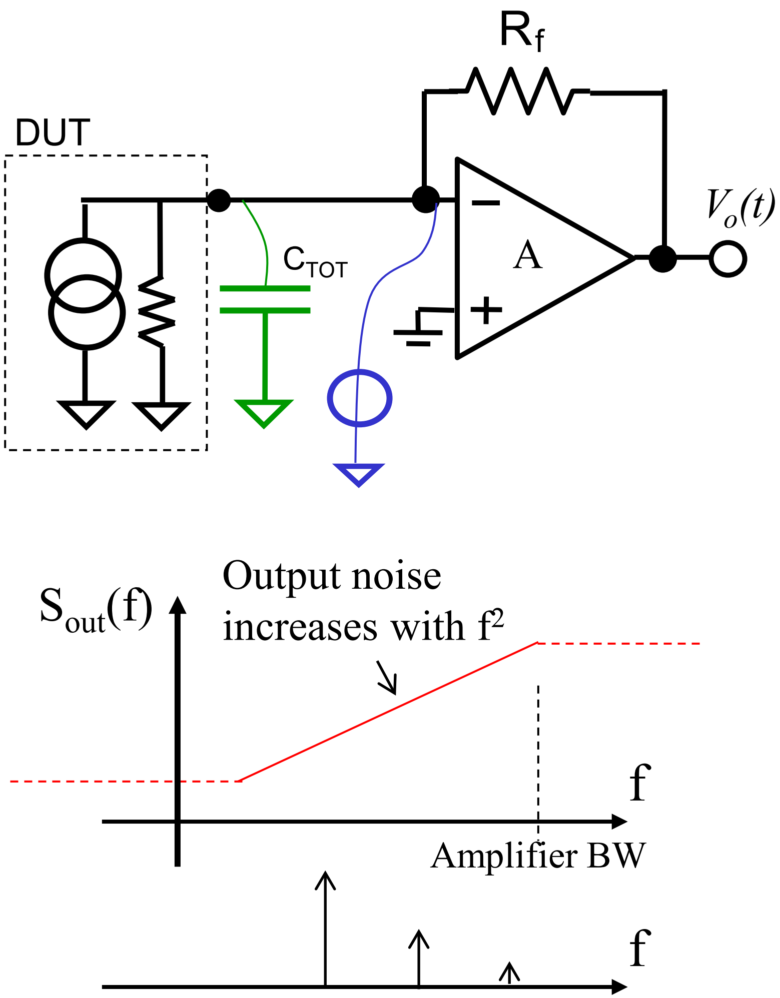
Improvement with Voltage amplitude



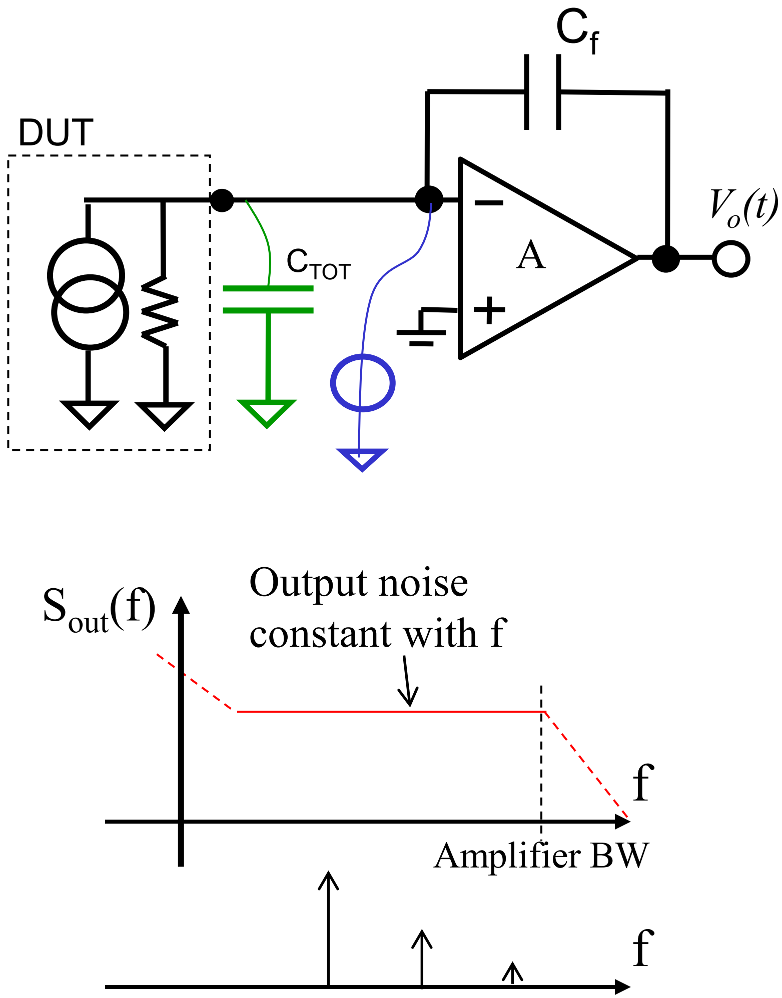




Special TIA for square wave mixer



If a square wave mixer is used, it introduces harmonics that fold a lot of noise



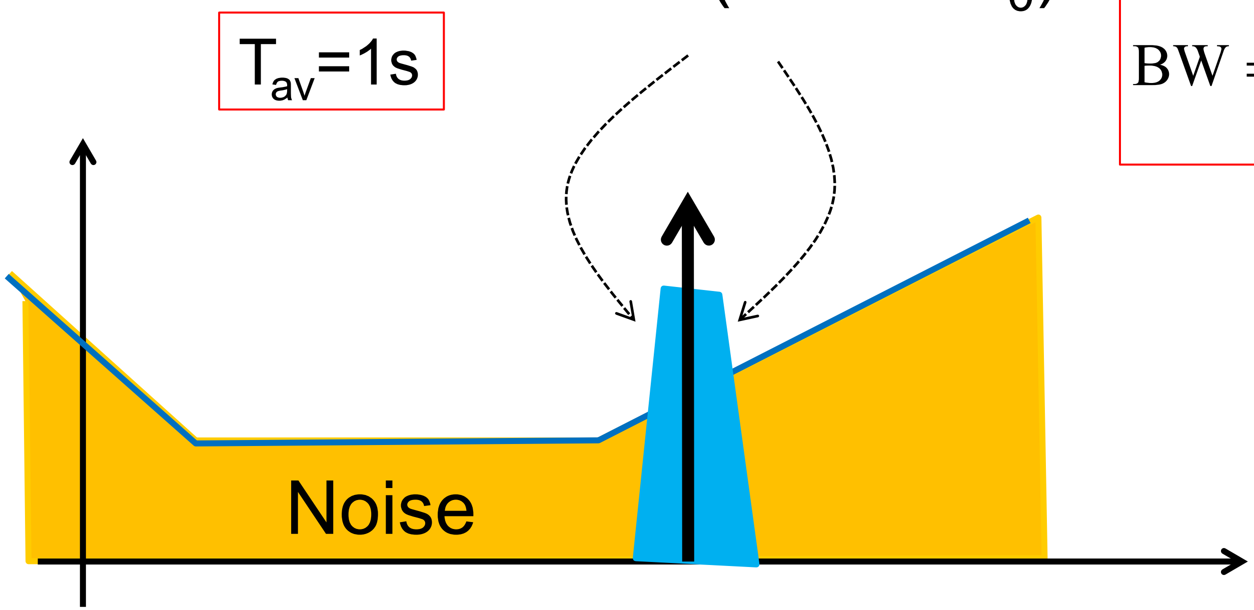
If a square wave mixer is used, higher harmonics give little noise

In conclusion …

Things to remember (1)

A Lock-in amplifier is «like» a band-pass filter at f_0

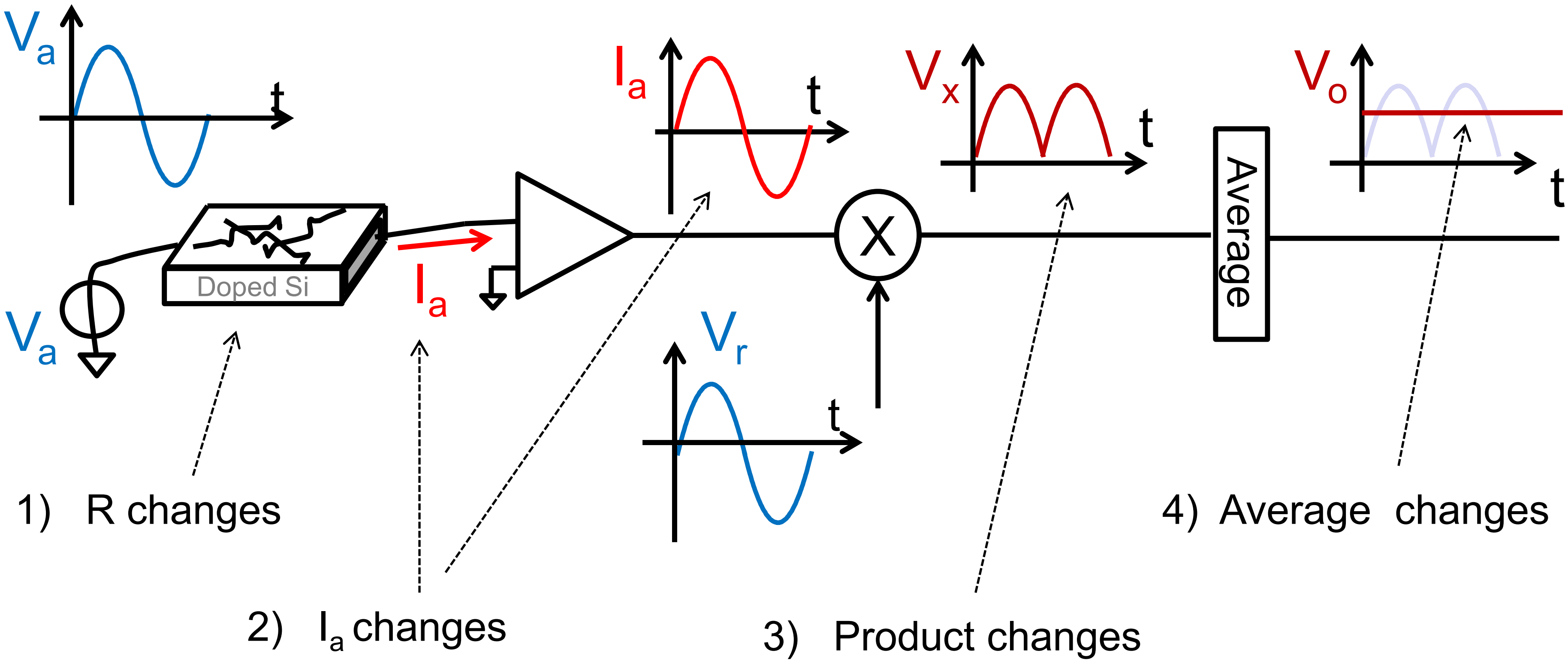
Longer averaging ➡ Thinner bandwidth (around f_0)



$$BW = \frac{1}{T_{av}} = 1\,Hz$$

By changing measurement frequency f_0 , you can choose the optimal position

Things to remember (2)



Signal variations can be tracked with time by simply monitoring the level of the output

Things to remember (3)

Extremely high sensitivity can be reached in device characterisation